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Evaluation of the Nonlinear Response Function and Efficiency of a Scintillation Detector Using Monte Carlo and Analytical Methods

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Abstract : A major advantage of Sodium Iodide scintillator, NaI(Tl), is its high efficiency in gamma ray detection. In this research, Using experimental values of Full Width at Half Maximum (*FWHM*), the response functions of a 4 in × Ø 4 in NaI(Tl) detector were investigated by MCNP4C code using GEB option as a special treatment for tallies. Computational results were compared with measured data by using standard gamma ray sources to check their accuracy. Then, the intrinsic efficiency was calculated for several source-detector distances and various sizes of NaI(Tl) detectors. Calculations were performed analytically as well as Monte Carlo method. It was found that the intrinsic efficiency has a minimum at d/R=0.8 (d is source-detector distance and R is the detector radius), independent of gamma energy. This minimum is justified by using the Dirac theory for the mean chord of photons in the detector.

Keywords: Gamma ray; Detection; NaI(Tl); Response function; Efficiency

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1. Introduction

The properties of scintillation material required for good detectors are transparency, availability in large size, and large light output proportional to gamma ray energy (Shafroth, 1967). NaI(Tl) scintillation detector is still widely used frequently in many applications (Gardner,1999; Metwally and Gardner,2004; He and Gardner andVerghese 1993; Khorsandi and Feghhi,2011).NaI(Tl) is still the dominant material for gamma detection because it provides reasonable gamma ray resolution and is economical. It has the advantages that it is efficient for high-energy gamma rays, it is rugged, and it can be used without cooling (Khabaz and Vega-Carrillo,2013). During recent years a large amount of experimental and computational works have focused on the study of response and efficiency of NaI(Tl) detectors (Miri-Hakimabad, *et al*., 2007; Gardner and Sood , 2004; Vitorelli*et al*., 2005). There are some factors which affect the detector response such as: detector dimensions, source-detector distance, and detector-collimator. In the present paper, response functions of a 4 in × Ø 4 in NaI(Tl) scintillation detector were determined using experiment and Monte Carlo simulations with the MCNP code (Briesmeister, 2000). After that, the intrinsic efficiency of different sizes of NaI(Tl) detector and its dependence on different geometrical parameters was evaluated based on analytical and Monte Carlo methods.

2. Response of NaI(Tl) detector

2.1. Experimental approach

Experimental setup which is used in this work is to take gamma ray spectrum from standard gamma sources. The experimental method for gamma rays from radioisotope sources doesn't need a complex setup. To obtain relatively good spectra, the gamma source is located 10 cm from a 4 in × Ø 4 in NaI detector while thedetector was insidea4.3 cm-thick leadshield for decreasing the background.

Using a Multi-Channel Analyzer (MCA) module and analyzer software, pulse-height spectrum of gamma rays were recorded in 1000-s time interval. During the spectrometer calibration, a linear relationship between the channel number, *Ch*, where the photo-peak appeared, and the photon energy, *E(keV)*, emitted by the calibrated sources was found. In the equation (1) this function is shown.

$$E(keV) = (-25.171) + (3.621)\ Ch \quad (1)$$

2.1. Simulation procedure

MCNP is a Monte Carlo N-particle code that can be used for neutron, photon, electron, or coupled neutron/photon/electron transport (Briesmeister ,2000)'

Tally 8 in the MCNP calculates the pulse-height. The pulse-height tally provides energy distribution of pulses created in a cell that models a physical detector. The F8 energy bins correspond to the total energy deposited in a detector in the specified channels by each physical particle.The initial responses of the MCNP simulation (pulse-height tally, F8) were broadened with the GEB option. Gaussian Energy Broadening (GEB) is a special treatment for tallies, to better simulate a physical radiation detector in which energy peaks exhibit Gaussian energy broadening.

GEB is called by entering FTn card in the input file of MCNP. The tallied energy is broadened by sampling from the Gaussian:

$$f(E) = C\ exp\left(\frac{E - E_o}{A}\right)^2 \quad (2)$$

where, E is the broadened energy; E_0 is the un-broadened energy of the tally; C is a normalization constant, and A is the Gaussian width.

The Gaussian width (Valentine, 1996) is related to the Full Width at Half Maximum (*FWHM*) by:

$$A = \frac{FWHM}{2\sqrt{ln\ 2}} \quad (3)$$

The desired *FWHM* that is specified by the user-provided constants, *a*, *b* and *c*, shows a nonlinear response:

$$FWHM = a + b\sqrt{E + c\,E^2} \quad (4)$$

where E is the incident gamma ray energy. The *FWHM* is defined as $FWHM = 2(E_{FWHM} - E)$,E_{FWHM} is such that:

$$f(E_{FWHM}) = \frac{1}{2} f(E_0) \quad (5)$$

here $f(E0)$ is the maximum value of $f(E)$.

Several standard gamma ray sources in the range from 0.059 MeV to 1.332 MeV were used to determine *a*, *b* and *c* as parameters specify the *FWHM* in the GEB option.

To calculate the amount of *a*, *b* and *c* parameters, the *FWHM* values obtained from experiment (Table 1) were fitted based on equation (4). These parameters were obtained as:

$a = -\ 0.00419 \pm 0.00223$ MeV; $b = 0.08591 \pm 0.00517$ MeV$^{1/2}$; $c = 0.20197 \pm 0.00887$ MeV^{-1}

Table 1 *FWHM* and resolutionvalues obtained from experiment for different gamma energies

E_γ *(MeV)*	*FWHM (MeV)*	[*FWHM (MeV)/* E_γ]×100
0.059	0.017	28.81
0.511	0.059	11.55
0.662	0.071	10.73
1.116	0.098	8.78
1.173	0.099	8.44
1.274	0.104	8.16
1.333	0.107	8.03

For ^{60}Co source, the directional response function was measured and compared with the simulated results. Fig. 1 shows a good agreement in the energy range of two photo-electric peaks and almost acceptable in Compton edge for the ^{60}Co gamma ray spectrum. The discrepancy in the lower energy region would be mainly due to the contribution of gamma rays scattered from the experimental photomultiplier.

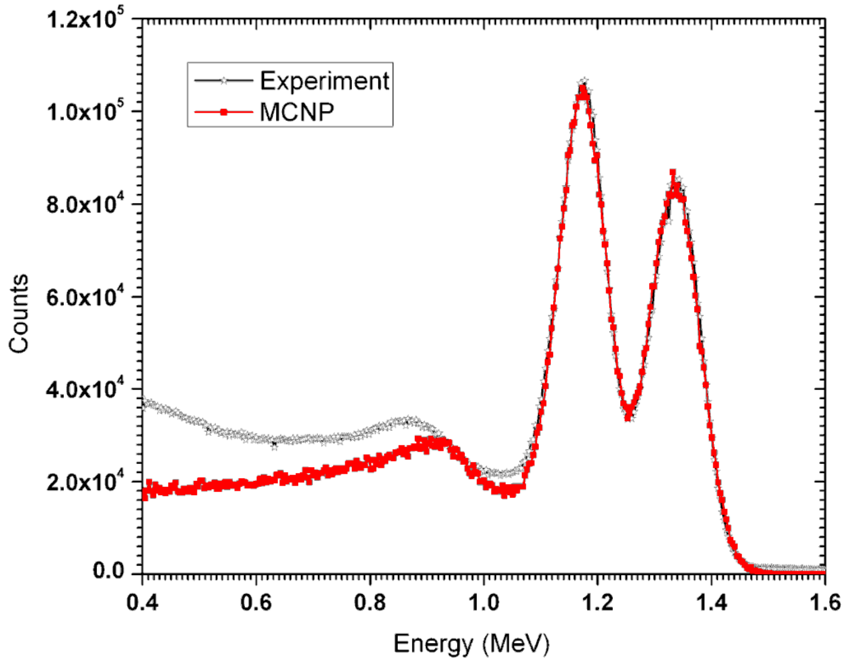

Fig. 1 Comparison between simulation and experimental gamma ray spectrum of the ^{60}Co

1. Intrinsic efficiency

3.1. Analytical method

Usually particles that enter the detector, they are not counted to be completely; in some cases, depending on the type and energy of the particle, size and type of detector, it may pass the detector without any interaction. Therefore, the intrinsic efficiency (ε_i) of the detectors is practically less than one. Intrinsic efficiency of the detector depends on the density and size of it, type and energy of radiation and electronic system connected to the detector.

If a collimated beam of photons with energy *E* is exposed to a detector with length of *l*, the probability of interaction a photon in detector is *1-exp[-μ(E)l],* which $\mu(E)$ is the linear attenuation coefficient of photons with energy *E* in the detector material. If an interaction is sufficient to produce a detectable pulse, the intrinsic efficiency of the detectoris calculated as

$$\varepsilon_i = 1\text{-}exp\left[-\mu(E)l\right] \qquad (6)$$

The intrinsic efficiency of the detector can be determined by measurement. For an isotropic point source with strength of *S* (Fig. 2), the intrinsic efficiency is

$$\varepsilon_i = \frac{r}{\varepsilon_g S} \qquad (7)$$

where*r* is the count rate by detector, and ε_g is the geometry efficiency which depends on the dimensions of detector and source-detector distance (*d*)

$$\varepsilon_g = \frac{1}{2}\left(1 - \frac{d}{\sqrt{d^2+R^2}}\right) \qquad (8)$$

here*R* is the radius of cylindrical detector. The absolute efficiency of detector is equal to the

$$\varepsilon_a = \varepsilon_g . \varepsilon_i \qquad (9)$$

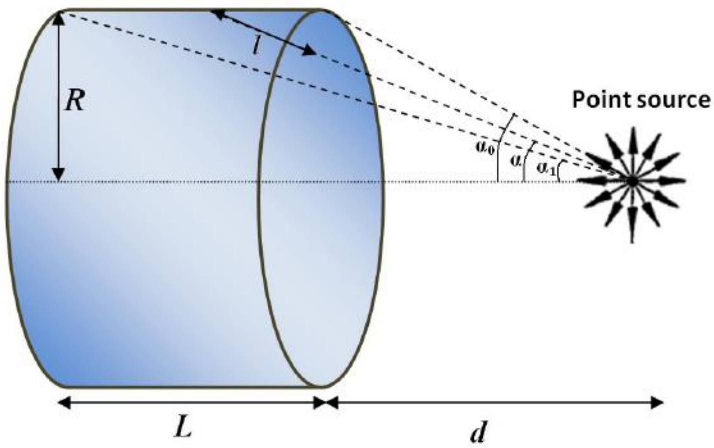

Fig. 2 A NaI(Tl) detector exposed to an isotropic point source

The intrinsic efficiency is also defined by

$$\varepsilon_i = \frac{\int_0^{\alpha_1}\left(1-exp\left[-\mu(E)\left(\frac{L}{cos\,\alpha}\right)\right]\right) sin\,\alpha\, d\alpha}{[1-cos\,\alpha_0]} + \frac{\int_{\alpha_1}^{\alpha_0}\left(1-exp\left[-\mu(E)\left(\frac{R}{sin\,\alpha}-\frac{d}{cos\,\alpha}\right)\right]\right) sin\,\alpha\, d\alpha}{[1-cos\,\alpha_0]} \qquad (10)$$

where L is the height of cylindrical detector; α_1 and α_0 are given by the following expressions

$$\alpha_1 = arctan\,(\frac{R}{L+d});\alpha_0 = arctan\,(\frac{R}{d})$$

Therefore, the intrinsic efficiency was determined analytically using equation (10Briesmeister 2000) for several sizes of a cylindrical NaI(Tl) detector exposed to an isotropic point gamma source at different distances.

3.2. Monte Carlo Method

One of the other methods having high accuracy for calculating the detector efficiency is the Monte Carlo method. The created pulse-height spectrum in various dimensions of NaI(Tl) detectors for several isotropic point sources with mono energetic gamma ray was calculated based on the MCNP4C Monte Carlo code using tally F8. For having a simulation close to real, photon and electron particles were tracked in the detector crystal. The number of counting in all energy bins and in photo-peak region was determined. After that, for each configuration, the intrinsic total and photo-peak efficiency of detector were assessed by dividing these counting numbers to the geometry efficiency.

Since, by MCNP may be modeled the geometry of a system, such as the NaI(Tl) detector, in detail, avoiding simplifications and approximations and also tracing different important particles such as photons and electrons in irradiation system while considering all of their interactions in detector, so the calculated detector efficiency will be similar to the actual efficiency.

4. Results and discussions

All calculations were performed for cylindrical detectors with equal diameter and height which have a relatively symmetric shape ($L=2R$). Fig. 3 compares the intrinsic total efficiency versus gamma energy achieved by analytical calculation and the Monte Carlo simulation, and also shows the intrinsic photo-peak efficiency obtained from Monte Carlo method for 3 in × Ø 3 in and 4 in × Ø 4 in NaI(Tl) detectors .

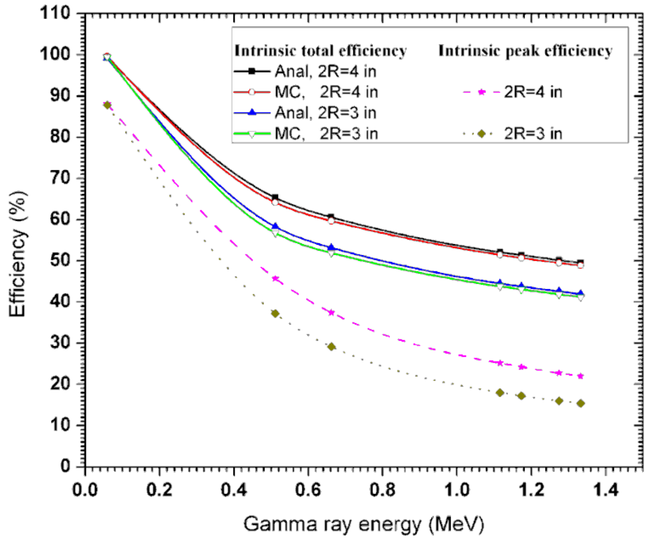

Fig. 3 Calculated intrinsic total efficiency and intrinsic photo-peak efficiency of 3 in and 4 in NaI(Tl) detectors (for *d*=*R*)

As can be seen, the larger detector has more intrinsic efficiency in all gamma energies, and for both with increase the energy, the efficiency decreases.

Furthermore, the ratio of photo-peak to total counts (photofraction) was shown for two sizes of NaI(Tl) detector as a function of gamma ray energy in Fig. 4.

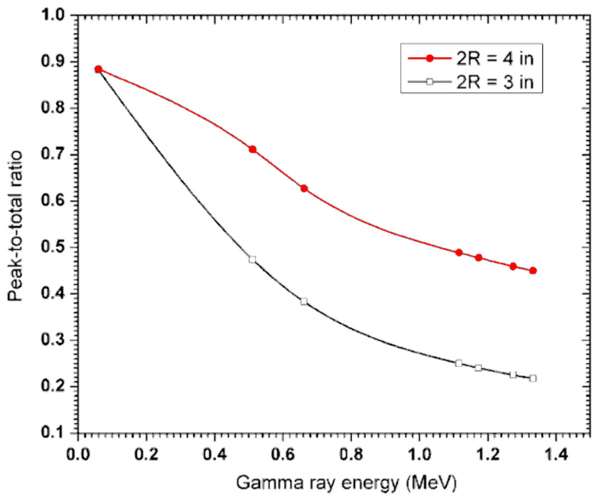

Fig. 4 Peak-to-total ratio (or the photofraction) for two cylindrical NaI(T1) for a point gamma ray source *d*=*R* from the scintillator surface

Total count in detector is related to all possible interaction of photon, i.e., Compton scattering, photoelectric effect, and pair production. It can be observed that with increase the energy, the photofraction decreases and, also for larger detector it is higher. This is because that for higher energy of gamma, the cross section of Compton scattering is more than photoelectric; and for larger detector, Compton scattered gamma rays and annihilation photons, interact within the detector volume and fewer of them escape from the surface.

The intrinsic efficiency of three sizes of NaI(Tl) detector and its dependence on different geometrical parameters (*d* and *R*) were evaluated based on analytical and Monte Carlo methods. Fig. 5 illustrates the comparison of intrinsic efficiency obtained from analytical and Monte Carlo methods for a 0.662 MeV gamma source as a function of *d/R*.

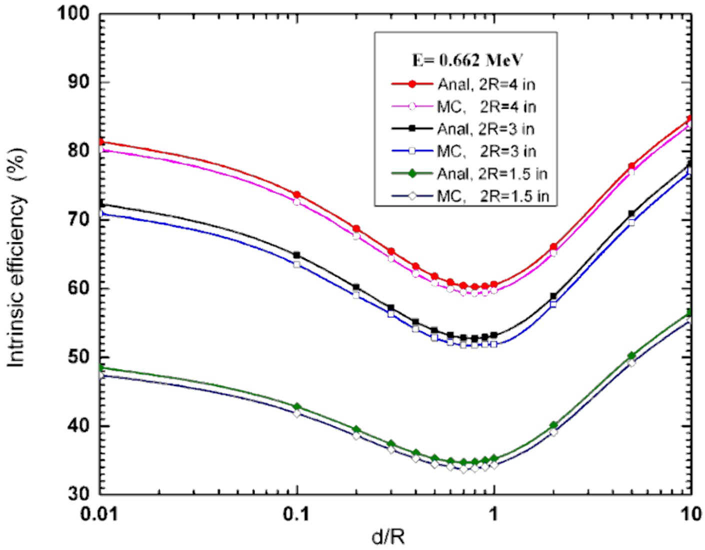

Fig. 5 Comparison between the intrinsic efficiency of different sizes of NaI(Tl) from analytical calculation and the Monte Carlo method for E_{γ}=0.662 MeV.

The results of analytical calculation and the Monte Carlo method are in good agreement; however, results plotted in Fig. 5 present minima of the intrinsic efficiency for all sizes of detector.Figs. 6 and 7 give the variation of the intrinsic efficiency with the ratio *d/R* for a 4 in × Ø 4 in NaI(Tl) at different gamma ray energies calculated by analytical and the Monte Carlo methods. As expected, efficiency increases when the gamma energy increases, since the less energetic photons have a greater probability of detection.

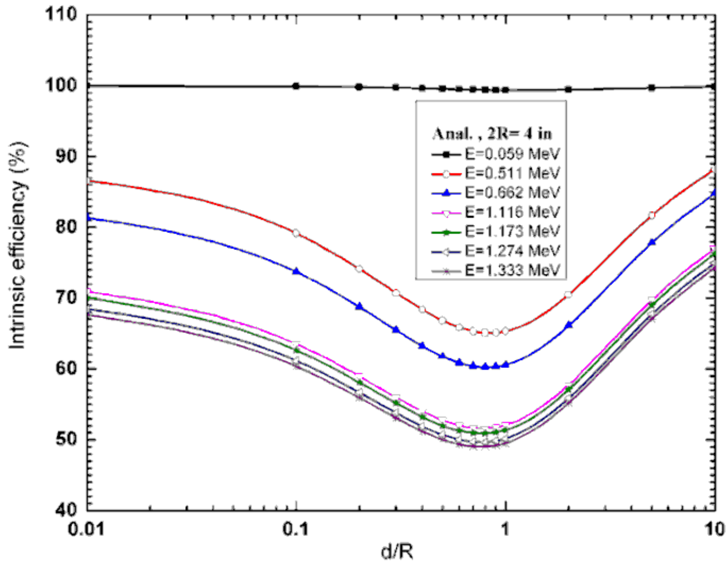

Fig. 6 Analytical calculations of intrinsic efficiency for a 4 in detector in different gamma energies versus ratio of *d/R*

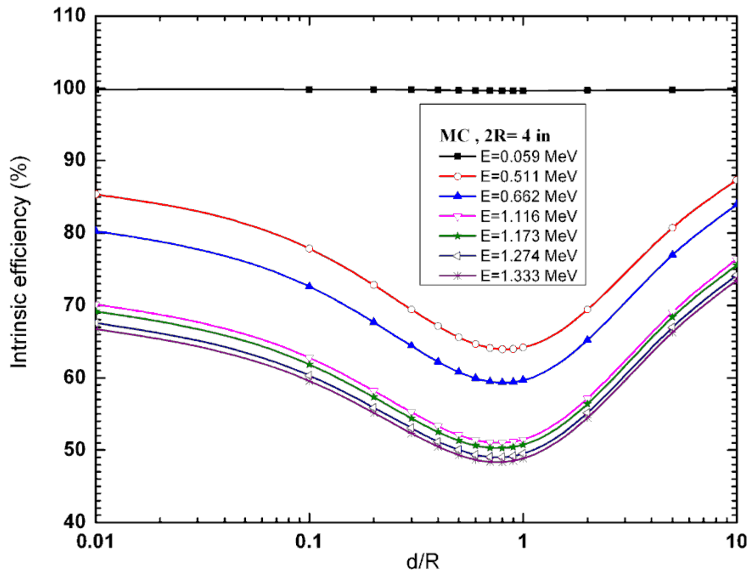

Fig. 7 Monte Carlo calculations of intrinsic efficiency for a 4 in detector in different gamma energies versus ratio of *d/R*

As can be seen, for all gamma energies, except 0.059 MeV, the intrinsic efficiency has a minimum at a certain source-detector distance. This minimum occurs in *d = 0.8 R*. This means that for a 4 in detector when gamma source is in distance of 4.06 cm from the surface of detector, the counting probability of a photon reaches to detector is least.

For explaining this phenomenon can be used the Dirac theory [12, Case *et al*., 1953]. According to the Fig. 1, length of the path traveled by rays through the detector varies between *0* and $L/cos(\alpha_1)$. For determining a mean length can use the Dirac theory. The mean chord, $\bar{l}$, of gamma ray in NaI(Tl) detector based on this theory is given by

$$\bar{l}(L,R,d) = \frac{1}{1-\cos\alpha_0}\left[L\, ln\left(\frac{1}{1-\cos\alpha_1}\right) + R(\alpha_0 - \alpha_1) - d.\,ln\left(\frac{\cos\alpha_1}{\cos\alpha_0}\right)\right] \quad (11)$$

With considering the certain dimensions for each case of *d/R* ratio, angles of α_0 and α_1 are specific values. According to the equation (11) and different values of *d/R*, the values of $\bar{l}$ were calculated, and results of them plotted in Fig. 8.

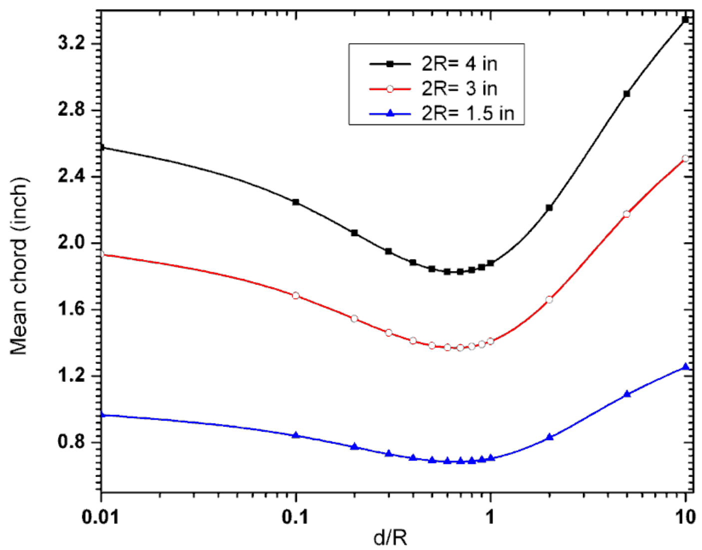

Fig. 8 Gamma ray mean chord in the different detector sizes as a function of *d/R*

As can be observed, mean chord for all sizes of detector has a minimum at a certain source-detector distance (*d = 0.8 R*). Therefore, obtained minimum in Figs. 5, 6 and 7 is due to the shorter mean path of ray through the detector.

For gamma energy of 0.059 MeV (in Figs. 6 and 7), the mean free pass ($1/\mu(E)$) is very shorter than mean chord in equation (11) in all detector sizes, and approximately all gamma rays reached the detector recorded.

5. Conclusion

Full width at half maximum (*FWHM*) from experimental results was used to evaluate the non-linear response function of a 4 in × Ø 4 in NaI(Tl) detector by using GEB option as a special treatment for tallies in MCNP4C. The results show that MCNP simulations by using GEB, with determined proper coefficients *a*, *b* and *c*, fit all the Gaussian peaks arising from standard gamma ray sources. It must be noted that GEB parameters are different for each configuration of experimental setup.

The intrinsic efficiency of NaI(Tl) detector was assessed using analytical expressions and the Monte Carlo simulation. It is concluded that the intrinsic efficiency is dependent on the source-detector distance, the dimensions of the detector and the incident gamma energy. It was shown that the NaI(Tl) intrinsic efficiency fall down at *d/R* ratios corresponding to the minimum of the gamma mean chord in the NaI(Tl) scintillator. Therefore, for having good gamma detection the source should be placed at a proper distance of detector.

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